MISSING DATA ESTIMATION FOR FULLY 3D SPIRAL CT IMAGE RECONSTRUCTION

Daniel B. Keesing\(^a\), Joseph A. O’Sullivan\(^b\), David G. Politte\(^c\), Bruce R. Whiting\(^d\), and Donald L. Snyder\(^b\)

\(^a\)Dept. of Biomedical Engineering, \(^b\)Dept. of Electrical and Systems Engineering, \(^c\)Mallinckrodt Institute of Radiology, Washington University, St. Louis, MO, USA

### Abstract

Reconstruction algorithms that are not set up to handle incomplete datasets can lead to artifacts in the reconstructed images because the assumptions regarding the size of the image space and/or data space are violated. In this study, two recently developed geometry-independent methods \(^1\) are applied to fully 3D multi-slice spiral CT image reconstruction. Using simulated and clinical datasets, we demonstrate the effectiveness of the missing data approaches in improving the quality of slices that have experienced truncation in either the transverse or longitudinal direction.

### Problem Statement

- When the support of an object lies partially outside the field of view (FOV) of a CT scanner, artifacts may arise in the reconstructed image due to undersampling.
- Most reconstruction algorithms implicitly assume the entire object is confined to the FOV, but if this is not the case, excessively large truncation values may be reconstructed inside the boundary of the FOV.
- The reconstruction algorithm is unaware of the measured data has been affected by the object’s attenuation outside the FOV, so the image in the FOV is reconstructed such that the projections through it match the measured data.

### Data and Image Spaces

- **(a)** Transverse view: the complete image space \(\mathcal{X}\) circumscribes the transverse support of the patient.
- **(b)** Longitudinal view: the complete image space \(\mathcal{X}\) is denoted by the gray slices.

### Existing Analytical Methods

#### Long object methods:

- Among existing methods, Schaller et al. \(^2\) developed an exact rebinning method called the Phi method, while Defrise et al. \(^3\) and Zou and Pan, \(^4\) have made use of differentiated backprojection along PI-line segments.

#### Transverse truncation methods:

- Among the extended FOV reconstruction methods, Hsieh et al. \(^5\) and Sourbelle et al. \(^6\) extrapolate the missing data in each projection using 2D parallel beam consistency conditions (e.g., constant area under projection curve in each view) and other constraints.

### Existing Statistical Methods

#### Long object methods:

- Zeng et al. \(^7\) published an iterative method that is similar in principle to the first approach of Snyder et al. \(^1\) (with one major exception being the choice of reconstruction algorithm). Rays that pass through both the ROI slices and outer slices are not used.

#### Transverse truncation methods:

- La Rivière described a joint estimation procedure that iteratively updates pixels and projections within the FOV. \(^8\) An initial estimate of the projections outside the FOV is obtained from a FBP reconstruction inside the FOV, and then subtracting its reprojection from the measured projections.

### Problem Formulation

We assume the photons arrive at the detector elements according to a Poisson counting process, \(\lambda(x)\), with mean value

\[
q(y ; c) = \mathbb{E}[d[y]] = \mathcal{L}(y) \exp \left(- \sum_{x \in \mathcal{C}} h(y|x)c(x) \right)
\]

where \(x\) = image voxel index, \(y\) = source-detector pair index, \(I\) is incident photon intensity, \(h(y|x)\) = projector kernel (mm), and \(c(x) = \) 3D truth image (mm\(^3\)).

### Alternating Minimization

#### AM algorithm (O’Sullivan and Benac) for monenergetic model

\[
\begin{align*}
\text{Patient} & \quad \text{CT scan} \\
\delta^k(x) & \quad \text{Forward projection} \\
\gamma^k(y) & \quad \text{Back-projection} \\
\beta^k(x) & \quad \text{Image update}
\end{align*}
\]

**where**

\[
\begin{align*}
q(y ; \beta^k) & = \mathcal{L}(y) \exp \left(- \sum_{x \in \mathcal{C}} h(y|x)\beta^k(x) \right) \\
\tilde{b}^k(x) & = \sum_{y \in \mathcal{C}} h(y|x)\gamma^k(y) \\
\gamma^k(y) & = \frac{\beta^k(y) - \beta^k(y)}{\sum_{x \in \mathcal{C}} \gamma^k(x)} \\
\tilde{c}^k(x) & = \max \left[ \beta^k(x) - \frac{1}{\mathcal{C}} \sum_{y \in \mathcal{C}} \gamma^k(y) \right]
\end{align*}
\]

### Missing Data Extension

From Snyder et al. \(^1\), the backprojections used in the image update step are different from the above algorithm as follows:

\[
\begin{align*}
\tilde{c}^k(x) & = \sum_{y \in \mathcal{C}} h(y|x)\beta^k(x) + \sum_{y \in \mathcal{C}} h(y|x)\beta^k(x) \\
\tilde{c}^k(x) & = \sum_{y \in \mathcal{C}} h(y|x)\gamma^k(y) + \sum_{y \in \mathcal{C}} h(y|x)\gamma^k(y)
\end{align*}
\]

Method 1: ignore missing data, \(i.e., \gamma^k = 0\)

Method 2: estimate missing data, \(i.e., \gamma^k \neq 0\)

The image space must be large enough to fully support the backprojections shown above, and all voxels in the image space must be updated to guarantee monotonic convergence.

### Transverse Truncation

#### Experiment 1 (NCAT phantom): Truncate the data from 3D detector elements on each side of sinogram (in all detector rows).

- Perform unregularized AM image reconstruction of 128x128x84 volume using specified methods.

- Experiment after 39 iterations

- All NCAT reconstructions shown after 100 full iterations with 73 ordered subsets.

#### Experiment 2 (NCAT phantom): Same experiment as above, except perform regularized AM image reconstruction of 128x128x84 volume using specified methods on noiseless and noisy data.

- A log cosh potential function was used to penalize differences between neighbors.

#### Experiment 3 (clinical abdominal scan): Truncate the data from 125 detector elements on each side of sinogram (in all detector rows).

- Perform unregularized AM image reconstruction of 512x512x176 volume.

#### Longitudinal Truncation

#### Experiment 4 (NCAT phantom): Perform unregularized AM reconstruction of 128x128x84 volume using specified methods.

- End slice initialization was done by replacing end slices after first iteration with the nearest fully-sampled slice.

- From left to right, the percentage of complete data for slices 76-81 was: 81.7%, 64.5%, 49.6%, 34.1%, 16.9%, and 4.5%.

### Conclusions & Future Work

- A feasibility study was conducted to apply two recently developed geometry-independent methods to fully 3D multi-slice spiral CT image reconstruction.

- The reconstructions using transversely truncated datasets demonstrate that it is possible to reconstruct the image inside the FOV quite accurately without many iterations.

- Outside the transversely FOV, some potentially minor artifacts are present. These artifacts diminish with increased numbers of iterations, leading to longer runtimes.

- Methods 1 and 2 addressed the long object problem comparably well when the end slices were initialized.

- It is possible that using some form of non-smoothing regularization, such as a prior based on consistency conditions, may significantly improve the convergence rate.

### Acknowledgments

This work was supported in part by a National Science Foundation Graduate Research Fellowship, by the National Cancer Institute under research grant R01CA75371 (J. F. Williamson, P. I.), and by the National Center for Supercomputing Applications (NCSA) with funding from the National Science Foundation and the State of Illinois. The authors wish to acknowledge the support of J. F. Williamson of Virginia Commonwealth University.

### References


### Corresponding author email: keesing@wustl.edu